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TM-1341
(SSC-N-22)
0102.000

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July 1985

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INTRODUCTION

Inside a kicker magnet, metallic beampipe cannot be used because it will screen off the rapid rising of the kickers's magnetic field. When a ceramic beampipe is used, one usually coats the inside with a thin layer of metal so as to carry at least part of the beam's image current and to prevent static charge buildup. The purpose of this article is to investigate whether such a coating will alter the risetime constant of the magnetic field significantly, whether such a coating can withstand the strong transient current induced by the fast rising magnetic field, and whether the back magnetic field generated by this transient current is strong enough to upset the designed risetime of the kicker.

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*Operated by the Universities Research Association, Inc., under contract with the U.S. Department of Energy.

The magnetic field inside the ceramic beampipe is affected by the metallic coating in two ways. Firstly, there is the "shielding effect". Even when the kicker's magnetic field rises abruptly as step function, the field inside the beampipe has a nonzero risetime

$$\tau_c = \frac{\mu b \Delta}{2\rho} , \tag{1.1}$$

where b is the radius of the beampipe, Δ the thickness of the metallic coating, ρ the resistivity of the coating and $\mu = 4\pi x10^{-7}$ henry/m is the magnetic permeability. Here, the azimuthal electric field and the radial magnetic field are assumed to be continuous across the coating. If we take Δ = 1 mil, b = 1.5 cm, $\rho = 1.7x10^{-8}$ ohm-m for copper, we get τ_c = 14 μsec or $\omega_c/2\pi = 1/2\pi\tau_c$ = 11 kHz. Thus, this is a low-frequency effect. The "shielding effect" has been considered in detail by Shafer[1] and will not be included in this article.

The second is the "skin-depth" effect. Here the fields "diffuse" from one side of the coating to the other. (The term "diffusion" is explained in the Appendix.) Thus, the azimuthal electric field and the radial magnetic field become discontinuous across the coating. This effect can be represented by the time scale

$$\tau_s = \frac{\mu \Delta^2}{2\rho} . \tag{1.2}$$

At the characteristic angular frequency $\omega_s = 1/\tau_s$, the fields will be attenuated by a factor of e^{-1} across the coating. Comparing Eqs. (1.1) and (1.2), we find $\tau_s/\tau_c = \Delta/b$. Thus the "skin-depth" effect occurs at a frequency much higher than that of the "shielding" effect. For a 1 mil coating with the same ρ, $\omega_s/2\pi = 6.7$ MHz. The injection and injection-abort kickers of the SSC have risetimes 10 ns to 100 ns, or have characteristic frequencies 1.6 MHz to 16 MHz. Thus, these kickers will be affected by the skin-depth effect, which we will study in detail below.

MAGNETIC FIELD ACROSS THE COATING

For simplicity, we assume the beampipe to be of square cross section with horizontal coatings of thickness Δ at the top and bottom walls as shown in Figure 1. The magnetic field generated by the kicker perpendicular to the coating is

$$B_{in}(t) = B_0 \left(1 - e^{t/\tau}\right) \theta(t), \tag{2.1}$$

where τ is the risetime of the kicker. This can be Fourier transformed into

$$B_{in}(t) = \int_{-\infty}^{\infty} d\omega \, B_{in}(\omega) e^{j\omega t}, \tag{2.2}$$

with

$$B_{in}(\omega) = -\frac{B_0}{\pi} \frac{\omega_\tau}{(\omega - j\xi)(\omega - j\omega_\tau)}, \tag{2.3}$$

where $\omega_\tau = 1/\tau$ is the characteristic angular frequency of the kicker and ξ is a positive infinitesimal number. The position of the pole near $\omega = 0$ has been carefully chosen so that the step function in Eq. (2.1) can be reproduced through the integration of Eq. (2.2).

Due to the time variation of the magnetic field $\vec{B}$, electric field $\vec{E}$ is also present. Inside the metallic coating, the corresponding Fourier components satisfy the Maxwell equations

$$\vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + j\omega\epsilon\mu\vec{E},$$

$$\vec{\nabla}\times\vec{E} = -j\omega\vec{B}, \tag{2.4}$$

where ε is the electric permitivity of the coating which we assume to be approximately the value at vacuum. The frequency of $\vec{B}$ and $\vec{E}$ for the kicker will be at most $\sim\omega_\tau/2\pi$; as a result, the displacement current can be neglected for most metal. Eliminating E, one gets the familiar equation,

$$\nabla^2\vec{B} = j\omega\mu\sigma\vec{B}. \tag{2.5}$$

The component of $\vec{B}$ perpendicular to the coating is continuous at both surfaces. Thus, after penetrating a thickness Δ, the emerging perpendicular magnetic field is

$$B_{out}(\omega) = B_{in}(\omega)\, e^{-(1\pm j)\sqrt{|\omega|\tau_s}}, \tag{2.6}$$

where τ_s is the characteristic time of the skin-depth effect and is given by Eq. (12). The ± sign in the exponent applies when ω is positive (negative). The time variation of the emerging magnetic field is therefore

$$B_{out}(t) = -\frac{B_o}{2\pi}\int_{-\infty}^{\infty} d\omega \, \frac{\omega_\tau}{(\omega - j\xi)(\omega - j\omega_\tau)} \, e^{-(1\pm j)\sqrt{|\omega|\tau_s}} \, e^{j\omega t} . \tag{2.7}$$

When $B_{out}(t)/B_o$ is viewed as a function of t/τ, the only parameter is

$$a = (\tau_s/\tau)^{\frac{1}{2}} . \tag{2.8}$$

When $t<0$, simple contour integration gives $B_{out}(t) = 0$ as expected. (See Appendix for detail.) When $t>0$, because of the existence of a branch point at $\omega = 0$, contour integration cannot be performed simply. Instead a numerical integration is attempted. The result is plotted in Figure 2. The rise of magnetic field inside the ceramic pipe at fixed kicker's risetime τ but at difference coating thicknesses can be read off directly. For example, with Δ = 1 mil and τ = 50 ns, i.e., a = 0.69, the effective risetime [the time for the field to increase to (1-1/e) of its maximum value] is roughly 3.5 times the kicker's risetime. If the thickness of the coating is reduced to 0.5 mil (a = 0.49), the effective risetime is still ~2.5 τ. Thus, the effect of the coating is not small at all.

We can also fix the coating thickness, and observe the rise of magnetic field at different kicker's risetime. Then we take t/τ_s as the time variable in Eq.(2.7) and take $a^{-1} = (\tau/\tau_s)^{1/2}$ as the parameter. This is plotted in Figure 3. We see that even when the kicker has zero risetime $a^{-1} = 0$, the field inside the coated beampipe has an effective risetime of $\sim 4.5\ \tau_s$.

EDDY CURRENT IN THE COATING AND BACK MAGNETIC FIELD

With the coordinate system in Figure 1, electric field E_y induced by magnetic field $B_z(t)$ at a distance x from the center of the coating is

$$E_y(x,t) = -x \frac{\partial}{\partial t} B_z(t). \tag{3.1}$$

As an overestimate, the magnetic field at the top of the coating surface is used, therefore

$$E_y(x,t) = -\frac{x}{\tau} B_o e^{-t/\tau} \theta(t). \tag{3.2}$$

Thus the current induced in one half of the coating is

$$I(t) \sim \int_o^W \frac{\Delta}{\rho} E_y(x,t)\, dx$$

$$= \frac{W^2 \Delta}{2\tau\rho} B_o e^{-t/\tau} \theta(t), \tag{3.3}$$

where 2W is the width of the coating. Taking W = 1.5 cm, B_o = 0.5 Tesla, $\rho = 1.7\text{x}10^{-8}$ ohm-m for copper, τ = 50 ns and

Δ = 1 mil, this current is ~1.7×10^6 amp when $t \ll \tau$. The total energy dissipated in a unit length of coating (two halves) is

$$\mathcal{E} = \frac{\Delta W^3 B_o^2}{6\tau\rho} = 4200 \text{ joules/m} . \tag{3.4}$$

This leads to a rise in temperature of

$$\Delta T = \frac{W^2 B_o^2}{12\tau\rho d s H} = 1600°C \ , \tag{3.5}$$

where d~9 gm/cm^3 and s = 0.092 are the density and specific heat of copper respectively and H = 4.18 joules/calorie is the mechanical equivalence of heat. In order to reduce this temperature rise, we have to choose a coating metal with high density, resistivity and specific heat. Nickel can lower the temperature rise by 5.4 times, wrought iron 6.4 times and steel ~50 times. Otherwise, some heat sink must be installed.

At the same time, the eddy current in the coating will generate a back magnetic field in the opposite direction of the kicker's field. At the center of the coating, this back field is

$$B_{back} \sim \frac{\mu}{4\pi}\int_{-W}^{W} \frac{2E_y\Delta}{x\rho}\, dx$$

$$= \frac{\Delta\mu W B_o}{\pi\rho\tau} e^{-t/\tau} \ , \tag{3.6}$$

which comes out to be ~180 B_o at $t \ll \tau$. Thus, special care must be taken for the kicker's current source so that the kicker's current will not be disturbed. Again, choosing a coating with a high resistivity can reduce this back magnetic field.

If the coatings are on the vertical walls of the beampipe instead, the rise in temperature and the strength of the induced transient magnetic field will be of the same order of magnitude, although there is no screening of the kicker's field at all in this case.

DISCUSSIONS

We see from above that a metallic coating will affect the transience of the kicker very much when the kicker's risetime is as small as τ = 50 ns. A coating of 1 mil copper can increase the risetime 3.5 times. The eddy current in the coating can lead to a temperature rise of ~ a thousand degrees. The back magnetic field can be ~180 times the maximum kicker's field when $t \ll \tau$. To overcome these effects we can choose a metallic coating with the highest resistivity. If we use steel whose resistivity is ~40 times that of copper, the risetime increases by only ~20%, the rise in temperature becomes ~32^{o}C and the back magnetic field becomes ~5 B_o. Such a high resistive coating may still serve the purpose of preventing static charge buildup.

To carry the image current some other methods must be derived. One suggestion is to use metallic strips at the vertical walls of the ceramic beampipe with capacitance in series. The capacitance is so chosen that it will allow the flow of the image current which is of high frequencies ($>\omega_{rf}/2\pi$ = 360 MHz) but block the flow of the eddy current which is of frequencies $<\omega_{\tau}/2\pi$ (= 3.2 MHz when the kicker's risetime is 50 ns).

The author would like to thank Dr. R. Shafer for useful discussion.

APPENDIX

Solution of Eq. (2.5) gives

$$B_{out}(t) = -\frac{B_e}{2\pi}\int_{-\infty}^{\infty} d\omega \, \frac{\omega_\tau}{(\omega - j\xi)(\omega - j\omega_\tau)} \, e^{\sqrt{2j\tau_s\omega}} \, e^{j\omega t} . \tag{A.1}$$

There is a branch point at $\omega = 0$ and we choose the branch cut along the negative ω-axis. For ω real and positive, the physical sheet is characterized by the physical condition that the integrand decreases as the coating thickness $\Delta(\sim\sqrt{\tau_s})$ increases; i.e.,

$$\sqrt{2j\tau_s\omega} \longrightarrow -(1+j)\sqrt{\tau_s\omega}$$

The same physical condition also applies when ω is real and negative; i.e.,

$$\sqrt{2j\tau_s\omega} \longrightarrow -(1-j)\sqrt{\tau_s|\omega|}$$

and is below the cut. The path of integration of Eq. (A.1) is shown in Figure 4. The pole-part of the integrand gives poles at $j\xi$ and $j\omega_c$ in the upper halves of both sheets. Therefore, when $t<0$, we complete the path of integration in the lower half of the physical sheet and get $B_{out}(t<0) = 0$. When $t>0$, if we complete the path of integration in the upper half of the physical sheet, we need to integrate around the cut too which is not easy at all. Instead, we do the integration directly by breaking it up into two parts: (i) along a semicircle of radius η in the lower half-plane to bypass the pole at the origin and (ii) from $-\infty$ to $-\eta$ below the cut and then from η to ∞. Finally we let $\eta \to 0$. The first part gives 1/2 B_o while the second part reduces to

$$B_2(t) = \frac{2B_o}{\pi}\int_0^\infty du\, \frac{e^{-au}}{1+u^4}\left\{\sin\frac{u^2t}{\tau}\left(\frac{\cos au}{u} - u\sin au\right)\right.$$
$$\left. - \cos\frac{u^2t}{\tau}\left(\frac{\sin au}{u} + u\cos au\right)\right\}, \qquad \text{(A.2)}$$

where $a = (\tau_s/\tau)^{1/2}$. Numerical integration then leads to the curves in Figure 2.

Equation (A.2) can also be written as

$$B_2(t) = \frac{2B_o}{\pi}\int_0^\infty du\, \frac{e^{-u}}{1+a^{-4}u^{-4}}\left\{\sin\frac{u^2t}{\tau_s}\left(\frac{\cos u}{u} - a^{-2}u\sin u\right)\right.$$
$$\left. + \cos\frac{u^2t}{\tau_s}\left(\frac{\sin u}{u} + a^{-2}u\cos u\right)\right\}, \qquad \text{(A.3)}$$

which gives the curves in Figure 3.

The transient electric field is given by Eq. (3.1). Using Eq. (A.2), we get

$$E_y(t) = -\frac{2\varkappa B_0}{\pi}\int_0^\infty du\ \frac{e^{-au}}{1+u^4}\ \frac{u^2}{\tau}\left\{\cos\frac{u^2t}{\tau}\left(\frac{\cos au}{u} - u\sin au\right)\right.$$
$$\left. + \sin\frac{u^2t}{\tau}\left(\frac{\sin au}{u} + u\cos au\right)\right\},$$

where a = 0 denotes the field on the top of the coating and a>0 the field inside or on the bottom of the coating. For t = 0, E_y = 0 independent of whether a = 0 or a>0. But for t = 0_+ we get

$$E_y = \begin{cases} -\varkappa B_0/\tau & a = 0 \\ 0 & a > 0\,. \end{cases}$$

Care must be exercised in the evaluation of Eq. (A.4) because the sin u^2t/τ term gives nonzero contribution with opposite signs when t = $0_\pm$. Equation (A.5) says that although there is a surge of eddy current on the top of the coating at t = 0_+, the eddy current on the other side of the coating always starts from zero no matter how thin the coating is. The same applies to B_z. If the kicker's risetime τ = 0, we get with the aid of Eq. (A.3),

$$B_z = \begin{cases} B_0 & \tau_s = 0 \\ 0 & \tau_s > 0 \end{cases}$$

at t = 0_+. This behavior can also be understood from the Maxwell equations. Both B_z and E_y satisfy Eq. (2.5), which in the time domain reads

$$\frac{\partial^2 E_y}{\partial z^2} = \mu\sigma\dot{E}_y\,, \qquad \frac{\partial^2 B_z}{\partial z^2} = \mu\sigma\dot{B}_z\,.$$

These are just diffusion equations. If there is a surge of fields at $z = 0$, it takes finite time for them to diffuse to the location $z \neq 0$. Thus no matter how small z is, the fields there always start from zero.

REFERENCE

1. R. Shafer, "On Shielding the Beam from Kicker Magnets at High Frequencies", this workshop. R. Shafer, Fermilab report TM-991.

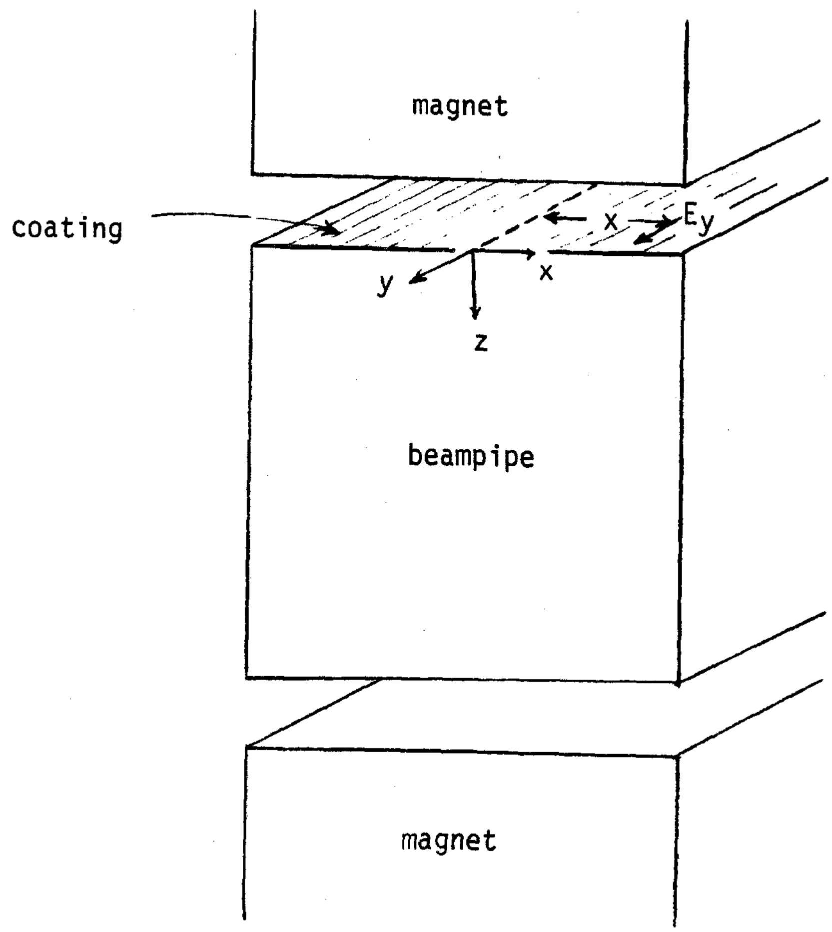

Fig. 1 Magnet and beampipe configuration

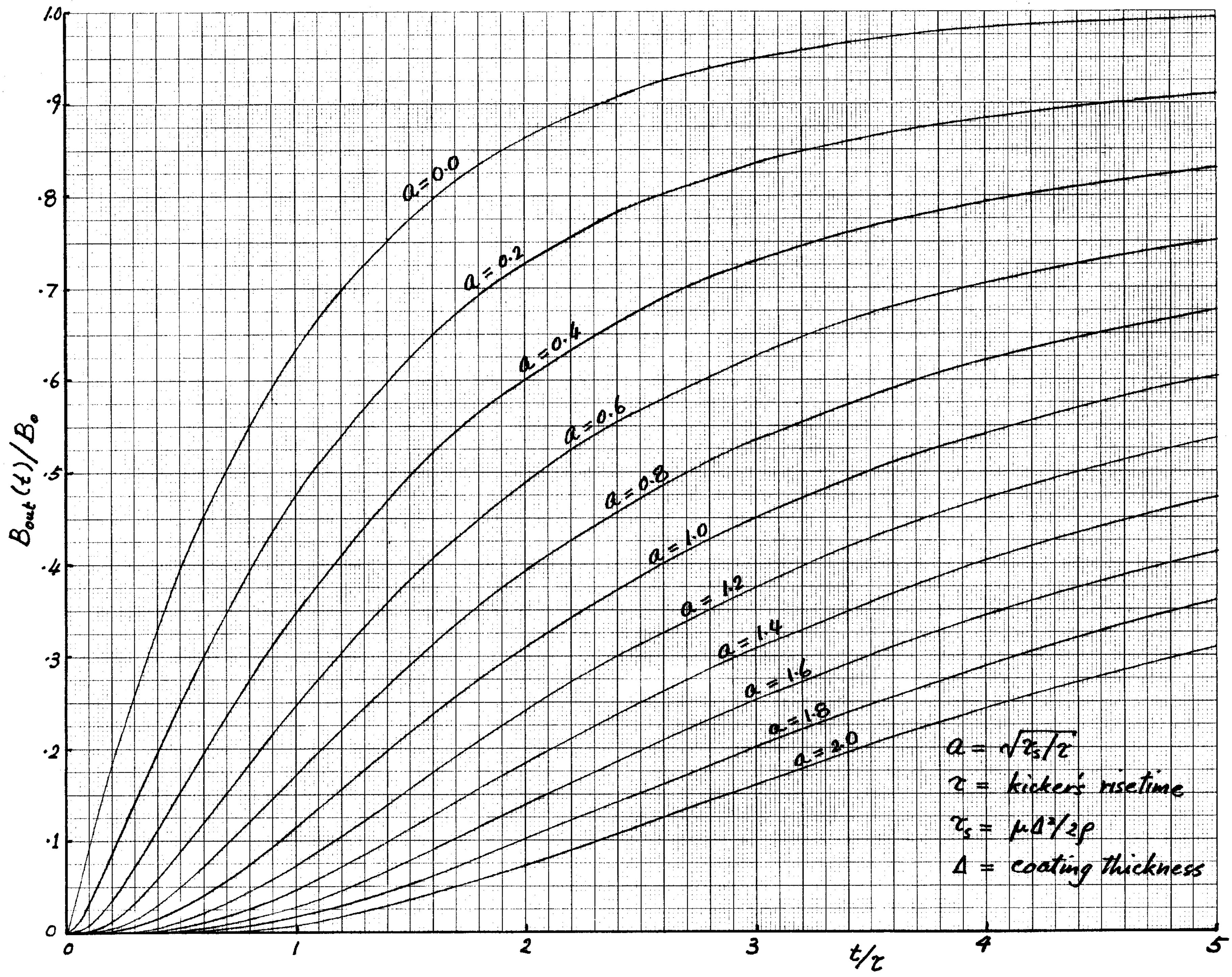

Fig. 2 Rise of magnetic field inside beampipe as a function of t/τ.

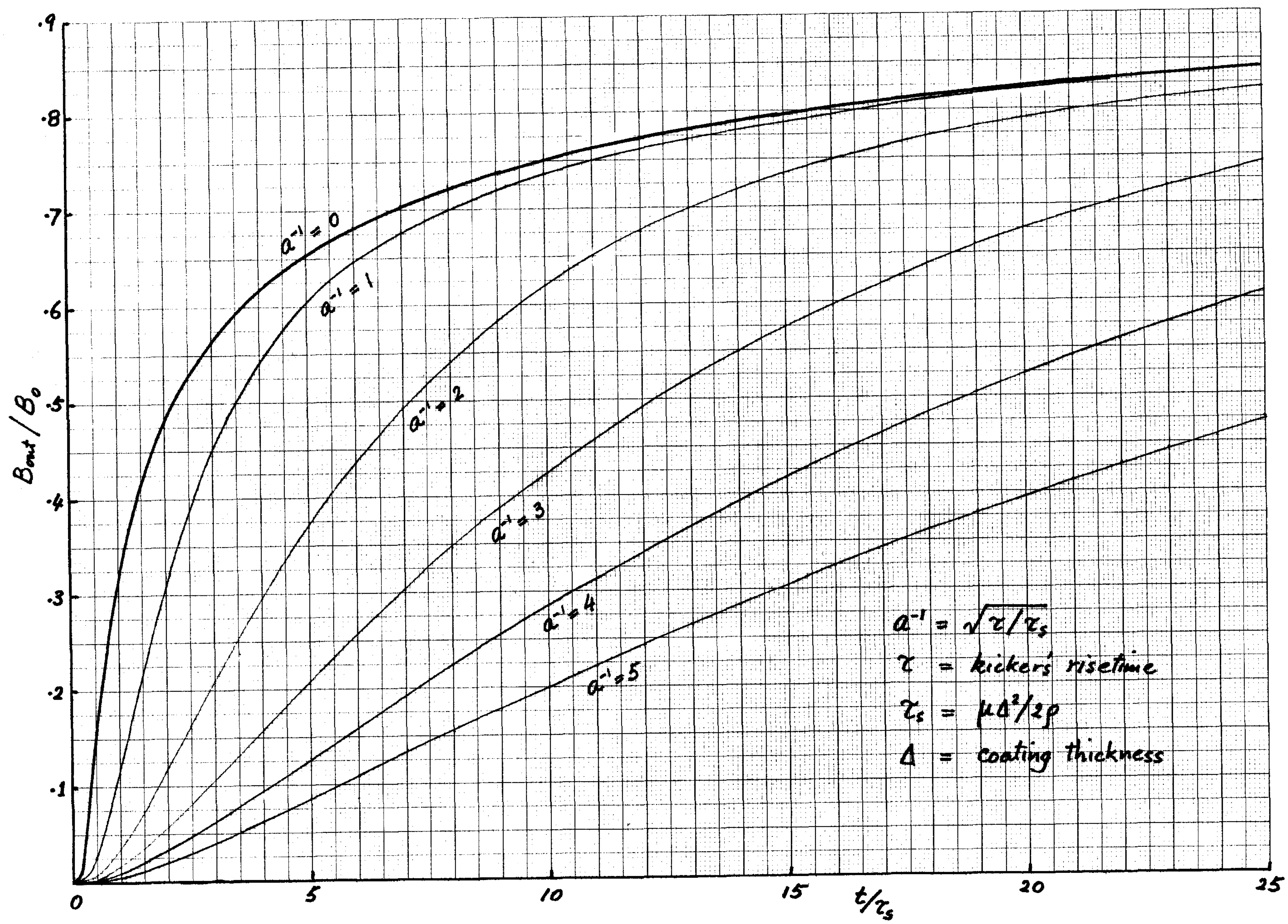

Fig. 3 Rise of magnetic field inside beampipe as a function of t/τ_s.

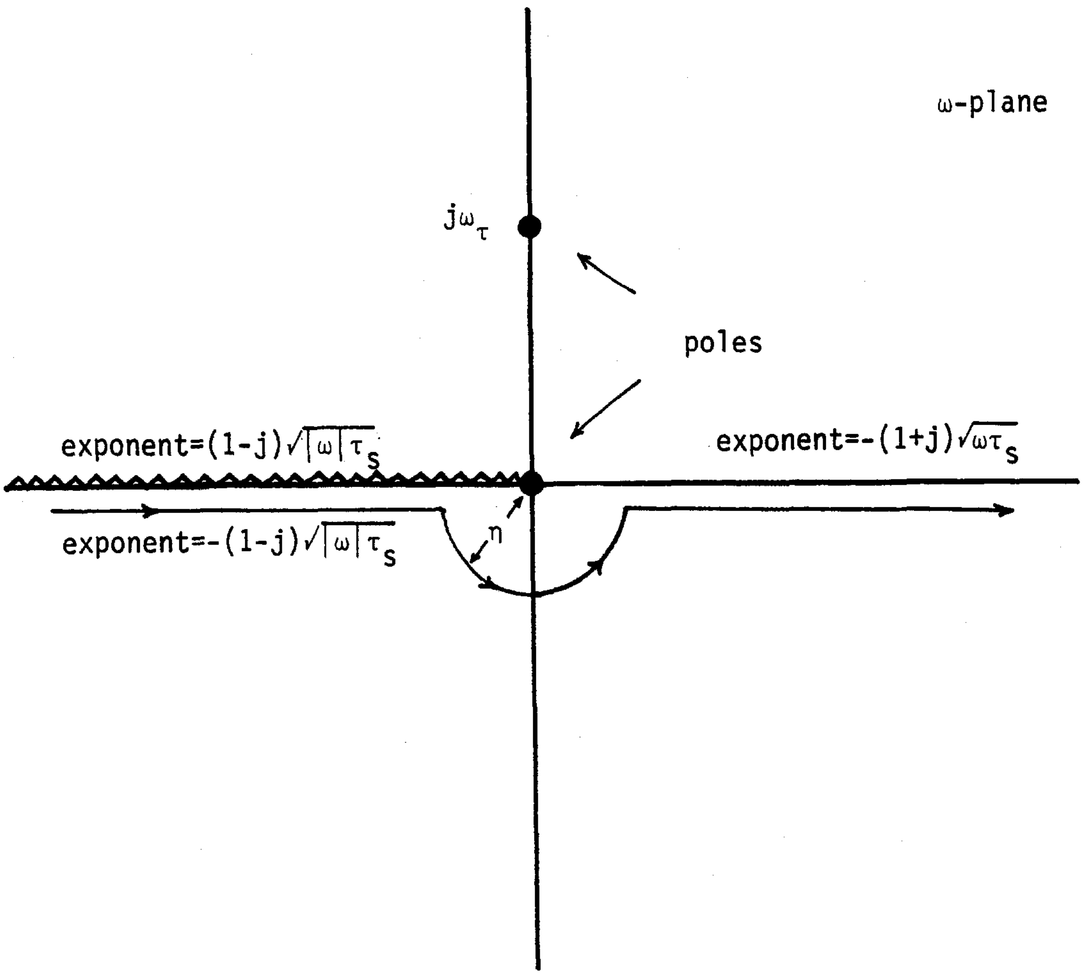

Fig. 4 The ω-plane and path of integration.